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[https://doi.org/10.1142/9789811247439\\_0010](https://doi.org/10.1142/9789811247439_0010)

## Chapter 10

# Laws of Form and Plato's Theory of Forms

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The remarkable sympathy of *Laws of Form* with Plato's theory of forms has been obscured by unsympathetic teaching of Platonic mathematical philosophy at our universities. Generations of students have sought the Platonic view only to be misguided by teaching that glosses, with incredulity and ignorance, Plato's project to found philosophy in mathematical formalism. Students often come away with

vague notions that forms are archetypes of things, where for example some particular horse expresses the ideal horse, where these model horses, tables, chairs and such have always hung out in Plato's ideal realm, as though moulds for the casting. Or worse, forms are common qualities of observed things whose independence from their observation ultimately comes down to correspondence with unobserved external material essences.

This is not the Plato of his dialogues, nor of the Renaissance, of Leonardo, Kepler or Leibniz. Rather, these are images of Plato that have prevailed since the so-called Enlightenment. Western philosophy of science since the beginning of the 18th century has become so deeply Aristotelian, and its host culture so thoroughly immersed in materialist ontology, that a fully formal philosophy can only be seen to deny the reality of things in an unscientific romantic delusion called "idealism." Yet Plato's historical position at the foundation of the Western tradition implies some obligation to teach him, and so two approaches have emerged at the universities. One finds Plato at the beginning of philosophy still in transition from *mythos* to *logos*, his mysticism naïvely trailing religious modes, the cleansing of which is our path towards modern science. The other approach finds sympathy with Platonic forms and their relevance to modern philosophy, but only by moderating the extreme view that sensible things are entirely projections of their formal essences, i.e., *Surely it cannot be forms all the way down!* A perhaps vague or unspoken presumption of ontological externality is retained so that Plato presents as proto-Aristotelian in the mode of Kant. This obscuring of Plato's mathematical approach means that Plato rarely comes to the attention of mathematically minded students. And when they discover in *Laws of Form* a new foundation for arithmetic providing a way out of Aristotle's logical bind, few would recognize its obvious Platonism. Indeed, Spencer-Brown himself might have missed this at first.

*Laws of Form* was conceived in the wake of an unresolved controversy over the foundations of mathematics, born to the school stridently rejecting mathematical mysticism for a logical ground. Spencer-Brown's discovery of the formal arithmetic underlying George Boole's *Laws of Thought* should be understood in this context. Boole's own discovery of the basic operations underlying Aristotelian logic, and then his application of arithmetic value to a new logical algebra, provided much of the elementary structure for this project to found mathematics in a logical hierarchy of classes (Boole,

1958). Its spectacular failure did not hinder the advancement at British (and American) universities of the neo-Aristotelian view it was supposed to serve (Lewin, 2018, pp. 31–42). This “Analytic Philosophy” was the context in which *Laws of Form* emerged. Which explains why it was long after this arithmetic was discovered, long after it gained the attention of Bertrand Russell, and only after *Laws of Form* was published, that its author would tell how he had just flipped us right back into Platonic mysticism (Spencer-Brown, 1972, pp. 108–109).

The flip back to Plato from Aristotle in this transition from *Laws of Thought* to *Laws of Form* is easily seen in terms of the relationship between *knowing* and *being*. If logic is a formalization of the language of our thoughtful means of *knowing*, and if our *knowing* comes down to thought about our experience of external material things, then epistemology does not involve its (external) ontology. Aristotle’s formal linguistics of *knowing* is ultimately about what it *is not*. What it *is not* is the external material *being* of its object. Aristotle developed this linguistic form/matter approach after many years a student at Plato’s Academy and in explicit rejection of the mathematical mysticism taught there by Plato and his collaborators. In the mode of the first *mathematici*, the Pythagoreans, these first Academics had no external reference because their epistemology involves its ontology, i.e., *Yes, it is forms all the way down!* And this is exactly what we come to in the final two chapters of *Laws of Form*: the object reduces to the subject, observed to observer, *being* to *knowing* and then *knowing* to the *being-of-the-knower*. All is one in form (Spencer-Brown, 1979, pp. 104–106).

The Platonic sympathies of *Laws of Form* can be awoken by following up Spencer-Brown’s own findings in Proclus and the “negative way” of pseudo-Dionysius, and then following these leads back to fragmentary accounts of so-called “Middle Platonism” and Plato’s first Academic successors. These sympathies are also found by going the other way, forward into Christian Platonism with Nicholas of Cusa, and then on to the advances in Platonic mathematics achieved by Kepler and Leibniz.<sup>1</sup> But neither of these paths will be taken

<sup>1</sup>For Spencer-Brown’s reference to Proclus’s *Elements of Theology* see Spencer-Brown (1979, p. 90), and to pseudo-Dionysius see Spencer-Brown (1972,

in this chapter. Instead, this is an invitation to find the Platonism of *Laws of Form* at its source, that is, in the writings of Plato. To do so, we must peer behind the modern caricatures of the theory of forms and rediscover the formal philosophy as introduced in Plato's dialogues and as so evidently informed by the arithmetic emanationism taught at his Academy. This effort is rewarded when Brownian notation is applied to Platonic arithmetic according to a hierarchy of its emanation preserved in an ancient Platonic textbook, which thereby reveals an order in the higher degrees of *Laws of Form* with direct application to geometry. Whether this interpretation of Platonic mathematics would have pleased Plato, Eudoxus and the other Academic mathematicians, or whether it was even anticipated by them, we can at best indulge in enthusiastic speculations.<sup>2</sup> But it is hoped that this Platonic approach to *Laws of Form* might light one path towards realising the vision of a complete mathematical science glimpsed some 50 years ago by George Spencer-Brown.

## 1. The Radical Nature of the Formal Approach

We seem to have this natural tendency to imagine that the independence of things in our sensible world is in their separate existence as distinctly observed. And yet all we know of sensible things is in our senses. To put it another way, I might say that all I know of observed things is in (or derived from) the observations of my observer-I, and so there is nothing that I experience or know that is not “internal,” i.e., it is all *in* my mind, *in* me. If all observation is entirely within the observer, then all sensible things must also be as they appear, which is present in experience.

Such a conclusion is common to many philosophical inquiries, as is a problem it presents. Observed things must be causally independent from the observer. Consider the proverbial tree. If it only exists in my sensible experience, then, when it falls, to presume I caused it to fall would be absurd, especially when I don't even observe the

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pp. 108–109). For a fuller account of Platonic sympathies, see Lewin (2018, pp. 174–295).

<sup>2</sup>For modern speculation on the evidence, see for example Taylor (1926), Weil (1957, Chapter 11) and Fowler (1999).

fall. Equally absurd would be to presume that other observers cannot see the same tree. One way to address the causal independence of our shared world is to propose that behind each sensible object is a material essence. This essential “thing in itself” would cause the sensible object to appear in *my* senses, and also in *your* senses. The *being* of sensible objects is transferred to an external material world, which exists in correspondence with the sensible world of each observer-I. The problem with such an external “ontology” is that it is unknown. The *being* of these external objects is beyond all sense of them, unknown and, by definition, unknowable. With sensible things reduced to appearances, and their *being* shifted into this unknowable realm, we cannot possibly know that they even exist. And so, this attempt to overcome the absurdity of causal dependence only presents another absurdity, which is that great embarrassment of modern philosophy sometimes called the mind/body problem.

No matter how philosophers have wrestled with the mind/body problem, they have never solved it, with every attempt at best only obscuring the problem of incompleteness in any external-referencing system (Lewin, 2018, pp. 38–52). Nevertheless, this correspondence theory prevails in the universities, where it is usually characterized in Kantian terms so that the unknown “noumenal” world sits behind the “phenomena” that we each immediately observe and naïvely regard as real. With no conceivable alternative beyond a scepticism doubting that anything real can be known, there is nothing for it but to save “objective” science by overcoming our epistemological isolation with an extraordinary leap of faith into the external realm. No such leap is required in Platonic formalism.

Whether in Plato's time there prevailed anything close to this Kantian dualism is not entirely clear. What we do know is that Plato took up a position explicitly against (if only) a naïve ontology of sensible things when he proposed that they are but shadowy projections of their formal essences. From the time of birth, according to Plato, we make sense of experience by generating objects of *knowing* through recognizing their form [*Phaedo* p. 75]. The ontological ground of experience is not in or behind the distinct objects as they appear. As formal constructions they only “participate” their formal *being*. This ontological source is internal. It is also insensible, undying and entirely outside time—characteristics usually associated with the

divine. Accordingly, knowledge of this reality is a kind of divination or “enthusiasm” (i.e., being filled with god) [*Phaedrus* pp. 242b–245c, 249b–e]. A path towards this divine wisdom is found through geometry and more specifically through arithmetic. The mind’s eye may start to see this reality by considering the essential nature of arithmetic. This is because the essence of things is like the essence of number. (Aristotle, 1933, p. 986). While numbered things are particular in their sensible appearance, number in itself is beyond any particular expression, insensible, timeless, immediately known. Just as two things participate absolute twoness, so does every other aspect of their *being* participate their formal nature. Before coming to what Plato sees as this formal nature exemplified by number, let us first consider the grand vision of his ontology as presented allegorically in perhaps his most famous dialogue, the *Republic*.

## 2. Allegory, Analogy and Ana-logia

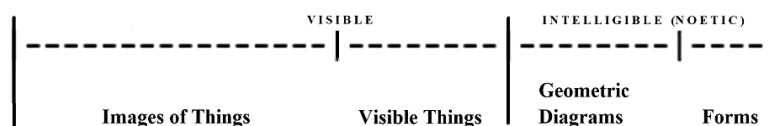
Good governance of Plato’s ideal state would be ensured if its philosopher-rulers have a clear view of absolute goodness. A lengthy program of education will aim to bring their “mind’s eye” to see it clearly. In this way, the allegorical account of forms is introduced to the discussion, as an attempt to show the object of this education: “the Form of the Good.” To see it is to know reality at its source. The sense of sight is the overriding analogy of Platonic enlightenment. The visible world is lit by the Sun-god, and so he plays the role of the supreme divinity:

*Just as* Sun *is to* things visible to the eye,  
*So* Good *is to* invisible forms as seen by the mind’s eye. (p. 508)

Just as Sun gives the ability to see things, so Good gives the ability to see forms. Plato expands on this imagery by introducing geometric proportion in the line. A line of arbitrary length is cut into two unequal parts and then each segment is divided again in the same ratio. This gives two pairs of segments in the same ratio on each side of the initial division to support the following analogy:

*Just as* shadows or reflections of visible things *are to* visible things,  
*So* geometric diagrams *are to* their forms.  
 (pp. 509e–510c)

Below is a line divided according to Plato's instruction, using the ratio of 2:1.



We are asked to first consider how things are resembled by their shadows and reflections. In the same way, geometric diagrams resemble their insensible geometric forms. When geometers draw squares and diagonals, their concern is not with the diagram's imperfect and particular visible existence. Rather, they use these images to explore the rules and relations of geometry in its ideal nature. This nature is their insensible formal *being* in what Plato calls the intelligible or noetic realm (*to tou nooumenou*). By noting that the diagrams are themselves visible things, Plato effects generalization to other visible things, including those whose formal essence is less apparent. Just as geometric diagrams express invisible forms, so too do all visible things. This reduction of the visible to the mathematical is expressed in the very geometry of our line, not that those ignorant of its geometry would ever know.

Below the playful dialogue of Plato's characters are often found layers of meaning, some vague and open to varying interpretations, others less ambiguous but esoteric. In this case, the adept might notice that the two middle segments of our line always measure the same, no matter in what ratio the first division was made. This implicitly reduces the analogy from a comparison of two ratios in a "4-term proportion" to a "continuous 3-term proportion" with a shared middle term:

*Just as images are to visible things so visible things are to forms.*

When finally we recall that sight is standing in for all the senses, our consideration of the insensible forms of geometry carries a much grander proposal, which is that all sensible things express their insensible formality. But what of the Form of the Good? The pretext of the whole analogy was its representation by Sun, but there is no place for the enlightening source on either side of the main divide.

This can be explained by Good's heightened mystical status. While the mathematical sciences lead most directly to knowledge

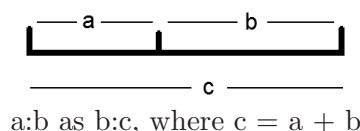
of forms, they are not the highest inquiry. The highest philosophical practice deals only with insensible forms in themselves as pure ideas without reference to diagrams, notation, images or otherwise to sensible things. Operating entirely within the extreme right segment of our line, this mystical practice can lead to the Form of the Good as the very first principle (*prantos archen*) (p. 511b). What this practice entails is not fully explained in the *Republic* but in other dialogues, which we will come to below. But the *Republic* has already told us just how elusive is its object. This is back in a prologue to this allegorical section, where a sideline to the analogy of sunlight notes that Sun not only gives visibility to things, but also their generation, growth and nurturing. Sun's primary role in biological generation serves to present the Good god as creator of ideal *being* as well as its *knowing*, and so beyond them both (p. 509b). That which is prior to the *being* of ideas cannot be an idea and so cannot be represented linguistically or otherwise. The ineffable and unknowable nature of the omnipresent omnipotent principle is presented with all due drama. The sage of our story, Socrates, is goaded into saying that he can only show what is "the offspring of the Good and what resembles it most closely" (p. 506e). In the following divide line analogy, it is geometry that most closely resembles the forms, which suggests a path to the first principle via the geometry of our analogy itself. Indeed, there are very Platonic reasons for paying closer attention to the undeclared ratio of our line's divide.

In the mathematical Platonism coming directly after Plato, geometric ratio (*logos*) was a means of emanation by continuous proportion (*ana-logia*). This is already suggested in Plato's dialogue *Timaeus*. Just after Timaeus declares that all living things come from one living *being*, this (fictional) Pythagorean cosmologist then considers the "bond" between them:

...the most beautiful [bond] is one that brings perfect unity to itself and the parts linked; and this is most beautifully done by *ana-logia*. (p. 31c)

Consider firstly the geometric double expressed in the series, 1, 2, 4, 8, 16, . . . . In this *ana-logia*, the "bond" is the *logos* of double, which is the means of emanation linking the series or "parts" with their unary origin. In such a generation there must be an origin (*arche*), in this case 1, and then the repeated application of the *logos*,

in this case double, which is explicitly express in the “first-born,” in this case 2. If the geometric triple is applied to a unit origin, then the *logos* is triple, and the first-born is 3. Such continuous proportions were found to be very powerful, yet they could not be the elementary form of creation from the first principle alone. This is because unary self-generation requires an immanent *logos*. Such a special unit-origin would be potent with its own means and product of generation.<sup>3</sup> On the face of it that is impossible. A *logos* that is not triple or double but singular, when applied to itself as 1, cannot generate any difference at all because  $1 \times 1 \times 1 \times \dots = 1$ . Some difference must be applied to give the differentiation. But then it would be both one and multiple, which is logically impossible and one of the reasons Aristotle rejected self-referencing mathematical emanationism for other-referencing materialism. (Aristotle, 1957, pp. 408b30–409a4; Lewin, 2018, pp. 49–50). However illogical, it is not geometrically impossible. A unary origin that is its own *logos* can be expressed by a very special division of the line. This was known long before Aristotle, and perhaps long before Plato—we don’t know when, if only because it was a well-guarded secret. However, this secret was sometimes cryptically revealed—as it is in the *Republic*. This has only recently been discovered by modern scholars. It suggests that our line should not be cut at random, but according to this very ratio that Euclid would later reveal as “division by extreme and mean.”<sup>4</sup>

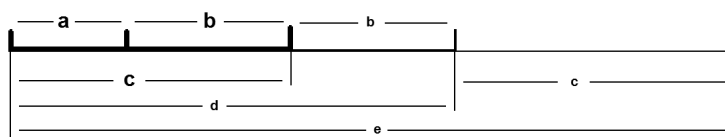


If a line is cut so that the extreme (smallest segment) is to the mean (largest segment) as the mean is to the whole, then the *logos* is

<sup>3</sup>For the self-generating principle (*arche*), see for example *Phaedrus* p. 245c–e. For creation from nothing, see *Sophist* p. 265c–6b, and in Platonic emanationism generally, see Lewin (2018, pp. 214–21).

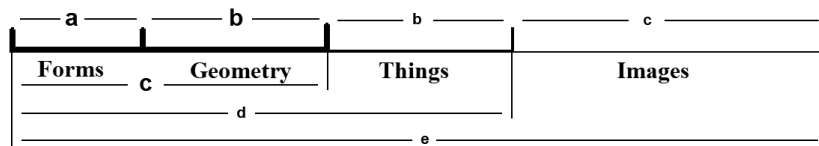
<sup>4</sup>A simple way to understand Plato’s cryptic reference is to imagine the *Republic* as one single line of text. Divide this line at the golden section and that finds the passage where the divided line is described. Bremer’s calculation by counting syllables (Bremer, 2000) is confirmed by Kennedy (2010, p. 22). There has long been speculation that the divide should be golden, and Kennedy gives reference to the previous speculations on cryptic evidence. Euclid’s instruction on how to make the cut is in *Elements*, Book VI. Prop 30.

in the whole, indeed, it *is* the line as so divided. If from this whole a geometric progression is generated by this same *logos*, then we have an *ana-logia* taking its “perfect unity” to all of its “parts.” Here is one way to present the first two stages of the geometric series according to this ratio that is now called “golden”:



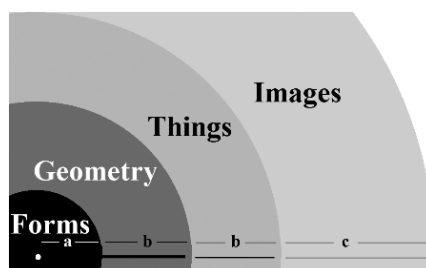
a:b as b:c as c:d as d:e

Notice that this emanation by golden ratio is like our depiction above of Plato’s divided line, only in mirror image. In fact, Plato does not specify whether the visible or noetic side is the greater, and among ancient Platonists cases are made for both options (Plato, 2013, p. 97, note 65). There is certainly some liberty for interpretation. What we know is that there is a hierarchy, that the *knowing* enlightenment ascends it towards the source of *being*, but the emanation itself is a descent to the lesser realities:



Plato’s line divided according to the Golden Ratio, where the first divide is  $c/(b + c)$  and the subdivisions are  $a/b$  and  $b/c$

The four categories of objects are here arranged from the left by the degree to which they contain a measure of reality (*being*), which is also the degree to which our knowledge of them is a measure of truth (*knowing*) (p. 511e). This hierarchy is better visualized in two dimensions.



The reader may be aware that ancient reverence for the golden ratio ( $\Phi$ ) has been vindicated by modern research into its elementary generative qualities, qualities that are found expressed in biological generation. It may also be known that  $\Phi$  is like the ratio of the square's diagonal to its side,  $\sqrt{2}$ , in that neither can be expressed as a finite arithmetic ratio. We will come to these two ratios again as elementary to the geometry developed below, where they find elegant infinite expression in the notation of *Laws of Form*. But if we return to the *Republic*, directly following the divided line is the famous allegory of the cave, and here Sun does make an appearance in all his brilliance. Ascent towards enlightenment begins with the discovery that sensible things are only shadows of real objects projected on the cave wall. Mind (*psyche*) then progresses out of the cave and into a realm lit by pure *being*. At first it can only observe shadows of the forms (cf. the geometric diagrams). Next, it sees the forms themselves. Finally, it braves the blinding brilliance and gazes directly at Sun. This leaves the reader wondering about the nature of this form of *being* and *knowing*, and its relationship with particular forms. (pp. 514–516b).

To find more about Platonic formalism, one good place to start is paradoxically with Plato's so-called "unwritten teachings." While the dialogues are seen as Plato's public or exoteric teachings, it is from the more advanced research at the Academy that Platonism evidently finds its roots. The fragmentary record of the earliest Platonists is one source from which scholars reconstruct the activities of the "Old Academy." For its mathematics, a key source is Euclid's *Elements of Geometry* as informed by its ancient commentaries. But perhaps the most direct source of Plato's esoteric teachings is Aristotle; only his accounts are decidedly unsympathetic. It is via Aristotle that we know something of Plato's only public lecture, which was advertised to be "On the Good." The natural expectation was that Plato would be speaking on the object of virtue. Instead, this was a lecture about mathematics, which concluded (after most attendees had departed) with the declaration that "the Good is one" (Swift Riginos, 1976, pp. 124–125). This is only one piece of evidence to suggest what is taken as given in early Platonism, which is, that the highest principle discussed in the *Republic* and elsewhere in the dialogues under different names is a principle-unity from which all else emanates. In Platonism, its most common name might be translated as the "one-alone" or the "singularity," but it has long since been transliterated as "Monad."

But in one sense its naming is inconsequential. In its pure potency it remains nameless, inexpressible beyond all particularity, and so only implied or referred to negatively by what it is not. Below its nameless heights, at the level of its first “offspring” much can be said. And much is said. And much is found not only in the second-hand accounts of the Academy, but also in what is of interest here, that is in the very dialogues themselves. While esoteric Platonism enlightens their interpretation, especially their mathematical interpretation, there should be no misunderstanding about what is explicit and carefully laid out in Plato’s surviving writings. In the *Republic*, directly following the allegorical section, and in the later dialogues, we find key aspects of Platonic formalism neglected in its modern teaching.

### 3. The Form of Opposites

An overriding narrative theme across many dialogues is virtue, including the virtuous man and the nature of the good life. When it is realised that the virtuous man is nigh impossible to find unless the society in which he lives is good and just, discussion shifts towards the nature of the good society; hence the *Republic*, and the need for its philosopher-rulers to train towards the ideal of goodness. It is in context of these psycho-social discussions that forms are introduced similarly in various places. Typically, Socrates directs discussion towards consideration of some ideal such as goodness, justice or beauty. He then notices that these great reference points for judgement in public life seem to be inborn, not derived from social life or otherwise from the senses in any direct way. And yet, when reflecting on their meaning, the discussants have difficulty pinning down a definition. This raises the question: Where do these ideas come from and what is their nature?

Such passages read as introductory to more advanced discussions of form, and some dialogues go on to a fuller exposition, as in the *Republic* and the *Phaedo*. Of those that do not, this might only be Plato’s narrative or didactic choice. Otherwise, their appearance in the Socratic dialogues of Plato’s “early” period suggests that his formal philosophy was not yet fully developed or not yet advanced beyond that professed by the historical Socrates. The extent to which either or both these possibilities are true, we may never know. Whatever the case, these passages have attracted a studious fixation in

Plato's non-mathematical reception, which seems to explain misunderstandings of his formal ontology as a set of archetypes, or a set of "universal" categories, or a qualitative classification of sensible objects according to linguistic predication. Such views limit the potential for a complete formal emanation. They should be rejected for this reason, and otherwise, just as they have been rejected generally since Aristotle. Of course, the first to reject them was Plato.<sup>5</sup> But a fixation on these passages only goes some way towards explaining the prevailing distortions of Platonic formalism among non-Platonists. What it does not explain is the most striking neglect of modern teaching, which is the way form involves opposites.

Whenever the dialogue pursues a definition of ideals like goodness, justice or beauty, every attempt fails to find solid semantic ground. Except in tautology. Except in contradiction. Perhaps because so obvious, or so trivial, the modern reader often fails to notice that each form is the same as itself and not its other. Every consideration of one ideal is in terms of its other: good with bad, just with unjust, beautiful with ugly, and so forth. Take for example what may be the earliest introduction to forms in the form of piety. This is in the *Euthyphro* when Socrates asks, "What do you say is the nature of piety and impiety?" He then launches straight in with:

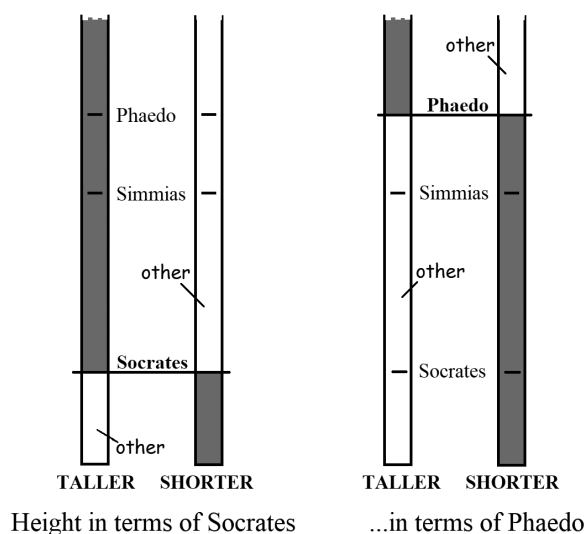
Is not piety always the same with itself in every action, and, on the other hand, is not impiety the opposite of all piety, always the same with itself and whatever is to be impious possessing some one characteristic quality? (p. 5d)

Tautology and contradiction. Same and other. A form is defined by itself and its negation. But this form of opposition is over and above the things that participate it. Of these things, both the one and its opposite may be present. A beautiful thing is never perfectly beautiful. Mixed in are aspects of ugliness. But absolute beauty can never be ugly. The one can never be its other, yet it cannot be without it "as if the two were joined together in one head." (*Phaedo* p. 60b)

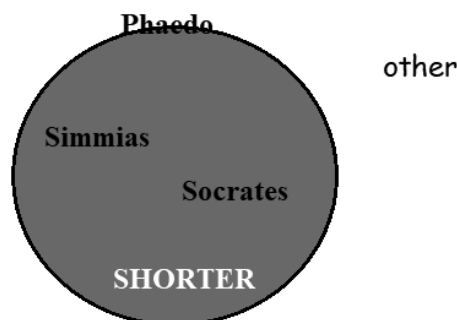
<sup>5</sup>Another less forgiving explanation is that the modern reception of Platonic formalism has been shaped by Aristotle's critique as found in his *Metaphysics* (Aristotle, 1933, pp. 990–992). Its "third man" critique is first found in Plato's *Parmenides* p. 132a–b. In medieval scholastic philosophy, the "realists" were those who saw "universals" as real and not just nominal qualities of things, which is a version of Aristotelianism characterized as Platonic.

In discussions of formal opposition, Plato gives emphasis to the elementary mensural relativity of greater/smaller. In the *Phaedo* (p. 102) Plato uses the heights of the characters in the dialogue to explain the participation of these two complementary forms in linear geometry. Simmias is greater in height than Socrates, but shorter than Phaedo, so Simmias participates both taller and shorter. This is not to say that taller itself is ever shorter, and yet the one implies the other.

Imagine a vertical line. Mark a divide called “Socrates.” There is no other metric. Call the part above “taller” so that any mark made above is in the taller space. Mark in “Simmias”. The first defining cut could be made anywhere. In this case, “Socrates” names the division that has “Simmias” participating taller. “Socrates” could also define shorter, only none present is shorter than Socrates! However, Simmias participates shorter under a divide called “Phaedo.”



Remember that this is only an example of an example. The reciprocal forms of greater and smaller can participate in other spaces, in plane space as area, or as solid volume, or as number, or scales for weight or for justice, or in any other applicable domain. And then greater/smaller is only one type of inequality. Sensible things participate form, some forms can participate other forms, but no form can ever participate its opposite.



Plato presents these opposites as a paradox of oneness that is also two. Taller is inconceivable without also shorter and *vice versa*. As perfect opposites they define each other in opposing each other, never overlapping or coinciding. A more general explanation comes in terms of likeness. Like things are like in some respects and unlike in others. These differences can be expressed in terms of equality/inequality or sameness/difference (the key Greek root here is “*iso-*” as in our “isosceles” or “isobar”). In every way that they are the same they are equal. As for their differences, these are all inequalities. But each of these inequalities is a sameness to itself. If the difference is shorter, then that is also itself a form. Thus in principle, a comparison between any two things reduces to the ways they are identical and different, which formally comes down to each way they participate in a form or its opposite, each of which is in turn a same/other unity equal to itself. This is the way that “all sensible objects strive after absolute equality” (*Phaedo* p. 75b). Anything and any difference that I may name or refer to is distinguished as one-not-other. We can now see how this conclusion informs the hierarchy of the sciences by returning to the *Republic*.

#### 4. The Unity of the Mathematical Science

Remember that we left off the *Republic* with the famous allegory of the cave. This was invoked to explain the absolute reality towards which the education program of the philosopher-rulers must be directed. Next comes an outline of this program in a hierarchy of the mathematical sciences. This starts with arithmetic, followed by geometry in the plane, then solid geometry and finally a fourth degree

geometry of moving solids. Arithmetic comes to the fore not only as the first science, but the one embracing all the others (p. 522b). As if this claim might surprise the reader, our sage Socrates declares extraordinary insight.

No one knows how to use [arithmetic] properly as something which in every way draws us toward reality. (p. 523a)

To explain this special power of arithmetic, Socrates abruptly changes course. We are asked to consider the aspect of sensible perception that requires higher intellectual (noetic) engagement. This is only when involving opposites. Socrates regards his own fingers in terms of their bigness/smallness, thickness/thinness, softness/hardness and lightness/heaviness. When considering one opposite, the other appears. To determine whether a finger is softer is to determine whether it is not harder. Each opposite seems to be both one (itself) and one of two (itself and not its other) (pp. 523c–534b). Each is indivisible and yet divided, separate and yet inseparable. The discussion is then brought back to unity as the principle and source of arithmetic. If the oneness of one thing (e.g., a finger) is viewed in an ordinary way by itself without opposition, then this is not what he is talking about. But when...

... something is always seen at the same time with its own opposite, so that it appears to be no more one than its opposite, it would immediately need some means of making the distinction and the *psyche* would be forced to be confused and to look for an answer thereby rousing thought within itself and raising the question *What is the number one exactly?*; and so the understanding of the number one would be one of those things which turn and lead it to the contemplation of reality. (pp. 524e–525a)

Let's now bring this all together. Everything is constituted by equal/unequal. The equality everything shares is its oneness. This oneness is the source of all arithmetic, the all-embracing science. It is a unity of opposites, where the one is never its other but is defined by it, in opposition to it, and in this sense also two. If the opposite of one is not-one, or none, then this principle-unity is difficult to express typographically but we will use the style "same(not-other)" and so "one(not-none)". Whether one(not-none), same(not-other),

equal(not-unequal), or by another name, the typography might be new, but the finding of this elementary form in Plato is no great novelty. On the contrary, just this form of opposites was taken up in the Platonism of Proclus, by pseudo-Dionysius and many others, including Nicolas of Cusa and right up to Leibniz's dyadic (or binary) arithmetic as an image of creation by alternation (Lewin, 2018, pp. 258–295). Arithmetic emanation as alternation is also to follow Plato, as we shall soon see. But the main reason to expect Platonic opposition at the heart of Platonism is not due to its elementary role in mathematics. It is also found elementary to a practice even more powerful than arithmetic, which is what Plato calls by the misleading name “dialectic.”<sup>6</sup>

## 5. Dialectic

Continuing again with the *Republic*, the education of the philosopher is not complete with the mathematical sciences. Directly after classifying music as a branch of fourth degree geometry, Socrates announces that all these studies are but a prelude to the main theme. Only through the higher practice of “dialectic” are the sciences brought into unity under the emanating first principle of everything. Mathematics is preliminary, but mathematicians are rarely dialecticians. After all, Socrates had just said that nobody knows how to use arithmetic “to draw us towards reality.” At least, nobody but Socrates! *So, Socrates, what exactly is this higher dialectic?*

Our first hint to the special role of dialectic come in the *Cratylus*, a short early Socratic dialogue about naming. Names are used to separate things according to their nature, and so good naming requires fidelity of the naming structure with the structure of reality. The right divisions of sensible experience can be found by asking and

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<sup>6</sup>Plato himself expresses concern about the name: “... whether the name I give to those who can do this is right or wrong, God knows, but I have called them hitherto dialecticians” (*Phaedrus* p. 266b). This is one passages that affords various interpretations. Did Plato regret first choosing this name when later refining the methodology? Or was he emphasizing, as elsewhere, that the assigned names are less important than the divisions named? A literal interpretation also holds, namely, that the true name is unknown but divine.

answering questions about the nature of things. The expert in this taxonomy is the dialectician. (pp. 389–390). Dialectic can be practiced at any level of knowledge, and we can safely presume that it is exemplified in the Socratic dialogues by every attempt to define ideals through their participations. But then in the *Republic* we have a dialectic operating exclusively in the noetic realm that can lead on to the form of forms.

We will recall *Republic*'s divided line as a hierarchy of known objects. To this is added a corresponding hierarchy of *knowing*, which we have already come to through the special role played by the mathematical sciences in bridging from the visual to the noetic realm, and as exemplified by geometric diagrams participating their forms. But the assumptions or “hypotheses” of geometry can only go so far. In leading to their forms, they occlude vision of absolute reality. Not so dialectic. It advances by “destroying” (*anairousa*) the mathematical hypotheses (p. 533c). In a higher discourse detached from all particularity, there can be “genuine hypotheses,” which “reason (*logos*) itself grasps by the power of dialectic,” and which can lead on to their “unhypothetical” first principle (p. 511b). This is no mean ask. Remember, this principle is beyond the *being* and *knowing* of all ideas. So too is this meta-science. If “the starting point is what is unknown,” if “the end and what comes between is woven together out of what is unknown,” then “what means are there that such a set of hypotheses can ever become knowledge?” (p. 533c) This very practice is a paradox. As for its methodology, there are few hints in the *Republic*. For that we need go to later dialogues where Plato presents his engagement with the ancient masters of paradox, the Eleatic sceptics. Through their attempts to destroy all pretention to knowledge Plato finds a way to the unknown source.

## 6. From Scepticism to Mysticism

The patriarch of Greek scepticism was Parmenides of Elea. His scepticism is presented by defending the claim “All is one.” In claiming “only one *is* and it cannot *not be*,” Parmenides refuses all science as self-contradictory because all claims of difference involve the claim that *non-being is*. Of course, the denial of difference and differentiation leads to many paradoxes, even in its very expression.

But that was the point. If his position is absurd then so are all others. Parmenides' Eleatic followers could always find contradiction in any claim for multiplicity or difference. Most famous was Zeno, and that was the purpose of his still-famous paradoxes. Zeno and his master were long dead when Plato was writing, but the sceptical onslaught that continued in varied and perhaps degenerate forms from the nearby city of Megara was difficult to ignore. Indeed, both Plato and Aristotle engaged. For Plato, this was especially and explicitly in two dialogues probably written soon after the *Republic*, first the *Parmenides* and then the *Sophist*. If their narratives are anything to go by, then this engagement was especially fruitful for Plato, as his higher dialectic is explained, demonstrated and promoted not by Socrates but by Eleatic sceptics.

Plato's *Parmenides* imagines a wonderful historical improbability, which is that a very young Socrates once engaged the ageing Parmenides and the not-so-young Zeno on a visit to Athens. The discussion opens with the youth boldly criticising Zeno's claim that if there are *many* then this involves a contradiction that the *like* is both *like* and *unlike*. Socrates frames his objection in terms of formal participation, much as we saw above. His argument is then challenged by Parmenides; whose questioning leaves Socrates completely disarmed. This exchange is often interpreted as an act of self-criticism, even that it announces Plato abandoning the theory of forms. There is not enough space here to address directly the problems with such an interpretation.<sup>7</sup> But if we continue our reading, it is Plato's fancy that the great Parmenides was impressed by a nascent theory of forms, but not its defence. Indeed, Parmenides does not challenge the existence of forms, only saying that the case for their sensible participation needs developing. Mastering the art of argument (*logos*) is key to finding the truth. "What is this art and its method of training?" By way of answering, Parmenides applauds Socrates' engagement with Zeno for avoiding reference to visible things and for staying within the formal realm. In this mode, Socrates should

<sup>7</sup>Those familiar with this now standard interpretation will see the criticism implicit in what follows. The arguments put into the mouth of Parmenides (pp. 130c–34e) mostly derive from presumptions that forms are less like number, more like things, and that their relations are of the form of linguistic predication.

follow Zeno in finding paradoxes, but not only by taking one side of the argument. He should practice by first framing a hypothesis, then consider what happens if it were true, and then all the consequences if it were false (pp. 135–136).

Of course, the finding of paradoxes by arguing both sides of a hypothesis would doubly destroy it. And so, this method is in accord with hypothesis destruction as advocated in the *Republic* where “reason (*logos*) itself grasps” absolute reality “by the power of dialectic.” Only here this is an unnamed method of *logos*. We have already crossed this term in the context of Platonic mathematics, where it had acquired the specific meaning of *ratio* as related to *ana-logia* as continuous proportion. Otherwise, *logos* has powerful resonances in the philosophical discourse of Plato’s time.<sup>8</sup> In the context of philosophical methodology, it often translates as “argument,” “reasoning” or “account.” Later, Plato will explain that, while both sophists and philosophers use *logos* to find relations, sophists work with appearances and false relations. Only true philosophers use *logos* in their dialectic pursuit of true *being*. Thus, there is some sense in having Parmenides (classed a sophist) advocate the philosopher’s method in terms of *logos*.<sup>9</sup> But the prominence of *logos* in these discussions around the emanating first principle also suggests that this usage informed the *logos* principle of generation that will appear prominent in Platonism, stoicism and Christianity. But what exactly is the meaning transmitted? Could it involve where Plato purposefully brought the mathematical *ratio*—as a principle of mathematical generation—towards union with this high discursive *rationalizing*? However purposeful or inadvertent this conflation of meaning, it is difficult to ignore in our interpretation of Plato’s formalism, which is (to use the Latin) that dialectic *rationalizing* finds the elementary *ratio* in the genera of *being* and *knowing* as expressing their formal genesis.

<sup>8</sup>The *logos* of Heraclitus is especially pertinent. For a survey of usage prior to Heraclitus, including where it already meant proportion, see Guthrie (1979, pp. 419–426).

<sup>9</sup>*Sophist* pp. 253c–254b & pp. 260–264. Despite Plato’s evident respect for Parmenides, in the *Sophist* his views are cast with those of his Megarian followers as sophistry, which, it should be said, only connotes pejorative due to Plato.

Anyway, when Socrates asks Parmenides to demonstrate his method of *logos*, we come to the main substance of the dialogue. This is Parmenides defending pairs of contradictory hypotheses: two pairs based on his claim that only one *is*; then another two based on its negation, that this one *is not*. With barely a reference to sensible things, this discourse on the Parmenidean “One” is thoroughly noetic, notoriously convoluted and exhausting to read as it cascades through many contradictory conclusions, some self-contradictory, including these:

- The One cannot be anywhere, for it could not be either in anything or in itself (p. 138a).
- The One is neither changing (*kineitai*) nor unchanging (p. 139b).
- The One cannot be either other or the same to itself or another (p. 139e).
- The One will neither be like nor unlike either other or itself (p. 140b).
- The One is not at all (p. 141e).
- The One is both one and many, a whole and parts, limited and infinite number (p. 145a).
- The One must be the same with itself and other than itself, and the same with all other things and other than them (p. 146a).

The dialogue closes without ceremony on this final claim:

Whether the One is or is not, the One and the others in relation to themselves and to each other all in every way are and are not and appear and do not appear.

What are we to make of this? For those who might balk at the mystical principle beyond elementary opposition, this multi-layered torrent of contradiction might present a spectacular refutation of Platonic formalism. However, in Platonism a very different view prevails. This may be approached by considering the affinity of the Parmenidean claim of no difference in “All is One,” with the Platonic unity of all difference by emanation. As we have seen, every unit of the emanation, its every difference, is found to participate the differential unity of the origin. And so, for Plato also, all is one. Thus, the whirls of logical contradiction around the *being* and *non-being* of the Parmenidean *one* might just be how “genuine hypotheses”

lead to the “unhypothetical,” unsayable principle. In fact, just such perplexing conclusions as those listed above would be taken up in a Platonic methodology which uses contradiction to attain the higher truth, that is, the methodology that came to be called “mystical theology” or “the negative way” (Dodds, 1928; Proclus, 1992).

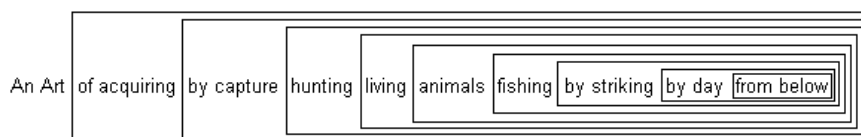
## 7. The Method of Division

In the *Sophist*, a dialectic method is explained and demonstrated that is much less cryptic and much less demanding on the reader. In fact, most of the narrative is decidedly playful, except towards the end where discussion shifts fully into the noetic realm as it approaches another perplexing and self-contradictory conclusion.

The *Sophist* opens with an unnamed Eleatic “follower of Parmenides and Zeno” asked to lead a Socratic-style discourse aimed at finding a true definition of that master of rhetoric and persuasion, the sophist. As various definitions are attempted, it comes to accidentally define the philosopher as practitioner of the very dialectic being demonstrated. This is the art of defining by right division (*diairesis*). As we already found in *Cratylus*, good naming should follow the separation of things according to their nature. In the *Sophist* and in another late dialogue, the *Phaedrus*, this dialectic method is said to have two aspects: firstly, to find the one form across separate individuals by “bringing together” (*synagoge*) scattered particulars; and secondly, to clearly define its limit. The art of finding the right division is compared with carvery, where the good carver finds the underlying joints before making the cut (*Phaedrus* pp. 265d–266d).

In the method of division, the first cut is made after choosing the subject to be defined and then looking for a general category to contain it. This category is then divided into two mutually exclusive forms or classes. The class containing the subject is divided again, and the process repeated until a class is defined containing only the subject. A quick demonstration is offered in defining the art of angling. Its very status as an art provides the first general category. The arts are then divided into those *of making* and *of acquiring*, where angling is an art *of acquiring*. Then the acquiring

arts are divided into those *by capture* and *by consent*. And so it goes, by one-sided dichotomous division as depicted here:



Each cut should divide by mutual exclusivity even where perfect opposites cannot be named. Thus, the division of the acquiring arts into *capture/consent* should conform to *capture/not-capture* precisely in the form *same(not-other)*. As the method proceeds, every new *same* becomes a nested *same(not-other)* (pp. 218e–221c). The recursive form is an analogue of the nesting of distinctions found in *Laws of Form* when the form “re-enters” one of its own inner spaces, as we shall see below.

After defining the angler, the Eleatic stranger leads on with the task of defining the sophist. This does not proceed so smoothly with a series of proposed definitions collapsing before they are fully stumped by the need to distinguish the sophist as one who says what is false. This would be to say what *is not*. Is that even possible? That which truly does not exist is contradicted by this very reference to it. How can we say the sophist speaks of what *is not* when it is inexpressible? (p. 238c) Our Eleatic guide turns attention towards his master’s admonition never to let the thought prevail that *not-being is* (p. 237a). The only way forward is to kill his master. This declaration for patricide heralds Plato’s answer to Parmenides. But it should not be missed that what is apparently required to permit the falsehoods of the sophist is exactly what Socrates refused in the *Republic* when pressed to give an account of the first principle of *being* beyond *being*, i.e., to speak of what *is not*. Now in the voice of a (newly enlightened) sceptic, Plato says that he will reveal the precise sense in which *non-being* exists and also how *being is not*, while still remaining silent about absolute *non-being* (p. 241d).

To show how Plato does this in the *Sophist*, we need to first go back to his *Parmenides*, where he showed that if there is a problem with the claim that *not-being* exists, then there is also a problem with the claim that *one is*. Eleatic scepticism worked by taking the

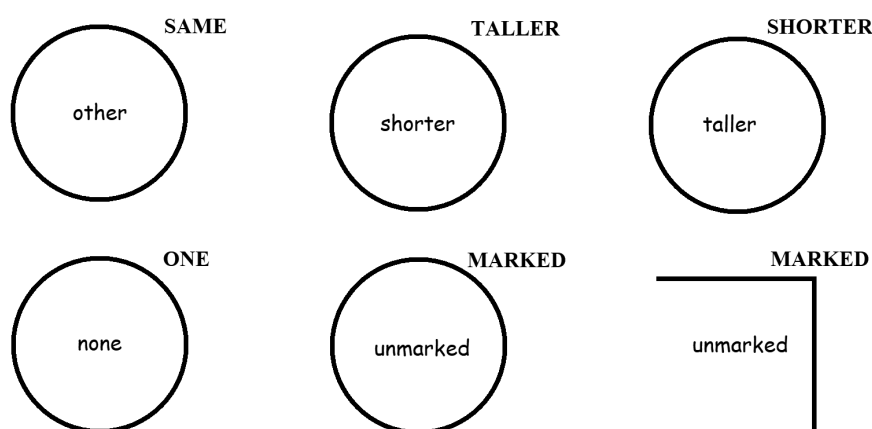
statement “Not-being is” to mean the predication “Not-being is existent.” In the same way, the statement “One is” can be taken to mean “One is existent.” In this statement, the idea of *one* is distinguished from its predicate, which produces two separate ideas, *one* and its attribute *being*. The question is then whether this *one* that is distinct from *being* also exists. In this context the question translates: Is this new *one* also a distinct idea? If we can refer to it, then surely it is. So, we can say of this new *one* that it is existent, which means that it also has the attribute of *being*. And so forth, every new *one* divides into *one* and its *being* so that the analysis fragments the original unity into a series of nested units of *being* by infinite progression (*Parmenides* p. 142). Essentially, Plato is generalizing the Parmenidean critiques to identify a fundamental insufficiency of linguistic method: its terms can only be grounded by reference to their meaning, but their meaning can only be predicated (or signified) with other terms.<sup>10</sup> Now, if we return to the *Sophist*, there Plato finds a way out of this logical bind through the form of opposites: the *not-being* of which we speak exists as *the other* of *being*. This is shown in two ways. Firstly, if we consider *being* as a form in itself, then any form participating *being* is in its own way *other* to it. Thus, all forms that are not the form of *being* are *not-being*, yet they still exist. The other approach is to consider that *the other* of any form is its *not-being*. Not beautiful is *not-being* beautiful; but, as *the other* of beautiful, it still exists. (pp. 257d–258a). Thus, if our sophist were to refer to the *unjust* as *just*, he is (falsely) referring to the *not-being* of *just*, which, as *the other* of *just*, also exists. With the roadblock removed, the defining game may recommence, only leaving in its tracks this brazenly illogical answer to the sceptics where “things that *are not* exist” and “the form of *not-being is*” because...

... the nature of *the other* exists and is distributed in small bits throughout all existing things in their relations to one another, and ... each part of *the other* which is contrasted with *being*, really is exactly *not-being*. (*Sophist* p. 258e)

<sup>10</sup>Just the same recursive form appears variously in linguistic and semiotic methods, include in Jacques Derrida’s analysis of the sign as “deferred difference” or “*différance*” (Derrida, 1982, pp. 1–27; Lewin, 2018, pp. 58–66 & 138–141).

## 8. Sympathies with Laws of Form

Hopefully those familiar with *Laws of Form* can see the sympathies here. Plato's dialectic division is the "distinction" of *Laws of Form*. It divides in the same(not-other) form, where *the other* is "unmarked." Opposite forms are the same distinction "marked" on opposite sides, perhaps alternating sides. But *the same* can never be its *other*, and so never marked simultaneously on both sides.



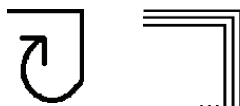
To mix up the language, we may say that from birth I generate my world by making distinctions of my experience. Each thing so generated arises through participating a great complexity of distinctly generated forms, some of which are more primitive, e.g., greater(not-less), some more archetypal, e.g., horse(not-not-horse), and some heavily reliant on synchronicity with other observers, e.g., sophist(not-not-sophist). Everything has its *not*, and so there are "small bits" of unmarked space "throughout all existing things in their relations to one another." In the notation of *Laws of Form* these unmarked spaces have the same value as the original unmarked page. Its first marking can represent the "first distinction" of the observer-I. If this represents *being*, then its *other* is *not-being*, which itself is a legacy of the unmarked place out of which observation first arose. This is Plato's *not-being* in the absolute sense, inexpressible, impossible to "indicated." If it were indicated then it would already have been distinguished as a particular distinction—which it was, in the first distinction! This first distinction, and all those following,

participate forms in themselves, which are unchanging, insensible, and ever-present outside time. The divine but intelligible (noetic) form of all possible experience is most truly visualized with geometric diagrams and arithmetic notation—as in the *Republic* so in *Laws of Form*, where the arithmetic notation itself is geometrically expressive. However, even these “hypotheses,” Plato warns, obscure the first principle of them all. Similarly, upon opening *Laws of Form* there is the quoted warning from Lao Tzu: the real source is unnameable. Only by destroying all particulars does all opposition resolve into absolute *non-being*. To this purpose are the paradoxes of the sceptics, which are used by Plato to reveal this origin of *knowing* in the *unknowing* of the unobserved observer.

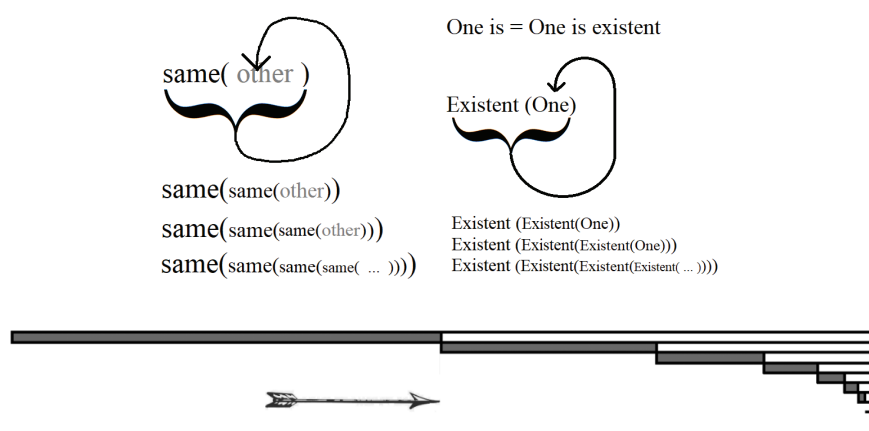
## 9. Reconstructing Platonic Mathematics Using Form Dynamics

So far, we have considered only Plato’s analysis of forms. This is the concern of *Laws of Form* in its first 10 chapters. These chapters advance on Plato with their “calculus of indication” arising from the laws of “crossing” and “calling,” which are only hinted at in Plato’s method of division with its “division” and “bringing together.” Otherwise, Plato has none of this proto-logical calculus of marked/unmarked states. The main way that such evaluation of expressions relates to Platonism is through generation by alternation of opposites. This may be compared to what has been called “form dynamics,” or the study of “re-entry” as first described in Chapter 11 of *Laws of Form* (Kauffman and Varela, 1980).

Re-entry is the recursive process initiated by a form entering one of its own spaces. The simplest re-entry is represented by a single mark re-entering one of its sides. It can re-enter either (or both) sides, but for now we consider only re-entry of its inner unmarked space. The effect of this “elementary” re-entry is that the mark appears inside itself, and inside again, so that repeated “crossing” generates an infinite series of nested marks.



In Plato, the perfect analogue of elementary re-entry is *the same* entering its other. We have already found this form in scepticism of linguistic method, where Plato turns the Parmenidean critique back onto the Parmenidean *one*. Taking the claim “One is” to mean “One is existent” turns the *one* into two distinct ideas. The new *one* again divides into *one* and *existent*. And so forth, the original *one* becomes an infinite nesting of *is*. Plato also notes the same process initiated by analysis of the predicate “existent.” This is *one* idea, and so the statement “Existence is one.” begins an infinite nesting of *one* on the side of the original *is* (*Parmenides* pp. 142b–143a). The same infinite form is found in the Megarian analysis of truth-value in the statement “I am lying.” If this statement is *true*, then it must be a lie, and so *false*. If I am lying that I am lying, then it must be *true*. And so forth, the statement self-negates its own truth-value. (Lewin, 2018, pp. 37–38). We don’t quite know whether this paradox was already circulating by Plato’s time, but we do know that he had Zeno’s critiques of mathematical physics involving dichotomous division on one side.



Draw a line across the page and divide it. Call the left division “past” and the right “future.” Divide the right division again into past and future, and again, however many times, the past will never fully traverse the line.

Enough has survived of this scepticism to show the currency of this infinite form in Plato’s time. Only, at the Academy re-entry

became elementary to positive methodology.<sup>11</sup> We have already seen its delimited expression in the method of division. Now we consider its infinite form as expressed in arithmetic. Only note that due to the brevity of this overview, the evidence justifying the reconstruction is not provided here but may be obtained elsewhere. (Lewin, 2018, pp. 170–266).

Prior to Plato, the Pythagoreans had a doctrine of generation from an unlimited origin (*apeiron*) by limitation. For Plato, this generation proceeds by alternation of opposites (*Phaedo* p. 70d–e). In arithmetic, the alternation of odd/even comes out of the original one(not-none) to provide an analogue of elementary re-entry:

$$\begin{array}{c} \overline{\cup} : \overline{\sqcap}, \quad \overline{\sqcap}, \quad \overline{\sqcap}\overline{\sqcap}, \quad \overline{\sqcap}\overline{\sqcap}\overline{\sqcap}... \\ \infty : 1^{\text{st}}, \quad 2^{\text{nd}}, \quad 3^{\text{rd}}, \quad 4^{\text{th}}... \end{array}$$

The alternation in marked/unmarked state of each successive expression corresponds to the odd/even alternation. In the Greek, “even” or perhaps “just,” is a good translation, but it helps to know that the word for “odd” means “excessive” [*perissos*] as in the exceeding of a limit. So the analogy has the marked state as excessive and the unmarked state as just, where just is not-excessive and excessive is not-just. By both accounts, this alternation may be seen as generated by *not-ing*, which answers a confusion in the ancient reception of Platonic arithmetic. Centuries after Plato, the received dogma was that the natural number series is the primary *ana-logia*. But this did not make sense because the successive terms are not in the same ratio, i.e.,  $\frac{1}{2} \neq \frac{2}{3} \neq \frac{3}{4}$ , etc. However, if we see the *logos* as division by *not-ing*, then it remains the medium of the whole series:

Odd *is to* even *as* even *is to* odd, where the common *logos* is *not*.

This is only to say that the ratio of opposite to opposite is the same whichever side you start, and we can now call this ratio “crossing.” Thus:

<sup>11</sup>At the Academy, infinite geometric forms were also used for  $\sqrt{2}$  and the “method of exhaustion” (Lewin, 2018, pp. 234–239).

Marked *is to* unmarked *as* unmarked *is to* marked, where the common *logos* is “to cross.” (Lewin, 2018, pp. 241–247)

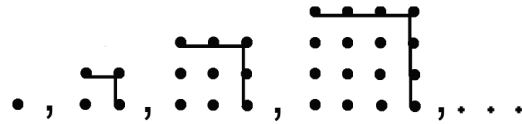
This elementary ordinal series proceeds in line and then develops by degree according to the geometric dimensions. In our conventional arithmetic there are no such degrees. These only come with the application of arithmetic to square-cubic geometry. Even then, our notation does not distinguish between measurements in different degrees. For example, a length of four and an area of four are both notated “4,” and equated accordingly. In the Platonic emanation, the dimensions build by motion in three degrees from an original point. When the point moves it generates the line. When the line moves it generates the plane. When the plane moves it generates the solid. The plane is not part of the solid but its origin. The line is the origin of the plane. The point is the dimensionless zero origin of all. In our conventional geometry and algebra, we might think of this emanation in terms of square space, where the unit in first degree is a line segment, in second degree a unit square, and in third degree a cube. However, in Platonic arithmetic and geometry another hierarchy prevailed, where the triangle is elementary in the plane and the tetrahedron is the first solid. A surviving handbook of Platonic arithmetic shows this hierarchy express by “figured” numbers as summarized in Table 1 (Nicomachus of Gerasa, pp. 237–238).

For this introduction we will only consider numbers up to second degree, where the first class is the triangular numbers, followed by the square numbers, pentagons, and so forth. This ordering is better

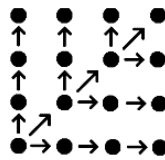
Table 1. The emanation of figured numbers according to Nicomachus (Nicomachus of Gerasa, 1926, bk. II Ch VII) (Lewin, 2018)®.

0-Dimension	<i>The point</i> corresponding to the original One or “Monad.”
1-Dimension	<i>The line</i> corresponding to the elementary just/excessive alternation i.e., [1], 2, 3, 4, 5, . . .
2-Dimensions	<i>The triangular</i> number series is elementary. From it are derived the square number series, the pentagonal numbers, the hexagonals, the heptagonals and so forth.
3-Dimensions	<i>The triangular pyramid</i> number series is elementary. From it are derived the square pyramid numbers, the pentagonal pyramids, the hexagonal pyramids, the heptagonal pyramids and so forth.

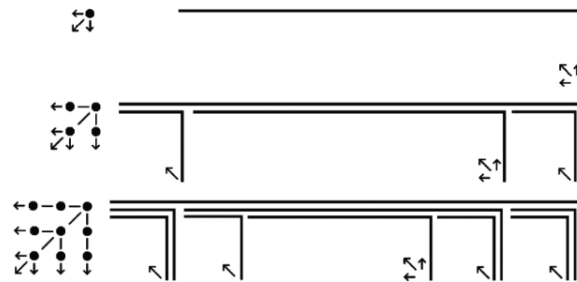
understood when expressed in Pythagorean dot notation. Consider firstly, the second class of plane numbers, the squares:




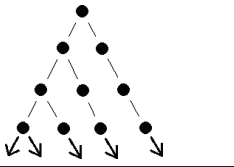


To mark the additions for each new square number, a builder's set-square, or "gnomon," was placed around the previous square. Thus, the generation of the square series [1], 4, 9, 16, ... builds by the gnomon series [1], 3, 5, 7, ... , and we can speculate that the path of generation is the simplest:



This is an analogue of the second degree re-entry of two elementary re-entries, notated thus:  $\overline{\overline{uv}}$ . If the origin is counted as the first period, then below we have the first three periods in both notations, only with the figured numbers orientated to support the comparison:



Notice how the dot progression equates with the nesting of marks. A hybrid notation is introduced below to support the analogy between unary second degree re-entry  $\overline{\square}$  and the triangular series:









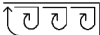
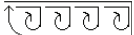
Pythagorean with speculated paths	Hybrid notation	Brownian notation as the expansion of 
		


In Table 2, an orderly presentation of the plane number series is seen to match an order in their Brownian notation by counting the number of marks re-entering at the second degree.

This table introduces derivative analysis. The “number series” is the total number of marks at each successive period. The “gnomon series” is the number of marks added at each period. The “gnomic interval” is the difference of the gnomon series. This third derivative presents the elementary number series, 1, 2, 3, . . . , which we will take as expressing the “simple infinite numbers” in second degree.

We are now venturing beyond ancient Platonic mathematics, but entirely in accord with it. Each figured number series is itself an infinite number. For example, the entire square series is 2 in the second degree of infinity. As for finite numbers, they are members of infinite series, and regarded as delimitations of their generation. In first degree, the number of periods corresponds to the number.


Table 2. The generation of plane figured numbers (Lewin, 2018).


	[Lines]	Triangles	Squares	Pentagonals	Hexagonals
The primary generation					
Expressed as Re-Entries					
Number series	1,2,3,4,5 . . .	1,3,6,10,15 . . .	1,4,9,16,25 . . .	1,5,12,22,35 . . .	1,6,15,28,45 . . .
Gnomon series	1,1,1,1 . . .	2,3,4,5 . . .	3,5,7,9 . . .	4,7,10,13 . . .	5,9,13,17 . . .
Gnomic interval	0	1	2	3	4


So, finite linear 4 comes with delimitation to the fourth period as .

This can be expressed as . This is saying that linear generation is taken to the fourth period.

Linear numbers can be expressed algebraically thus: .

In second degree, 4 comes with delimitation of the square series to the second period as .

This can be expressed as . This is saying square generation it taken to the second period.

Square numbers are expressed algebraically thus: . This means to take the square series to the  $x$ th period, i.e.,  $x$  is the number to be squared. This expression will be important for ordinary square-cubic geometry in the plane.

Notice how we now have two expressions for conventional 4, one as a square number,  $2^2$ , and the other as a linear number,  $1 \times 4$ . Two linear twos also give 4. However, in this arithmetic these are not equal. This can be explained in terms of the difference between Platonic arithmetic and its derivative art of “logistic.” *Logistic* means our ordinary counting and calculation. It counts things in one-to-one correspondence with the ordinal number series, for example, the counting of cattle by moving beads on an abacus for every cow passing orderly through a gate. (*Republic* p. 522c–d; Health, 1921, pp. 13–15)

In this way, marks within expressions can be counted. All these expressions have a count of four marks, and so we say they have “tally similarity.” Algebraically, this similarity is expressed using “tally” as a prefix (meaning “to be tallied”) and as a suffix (meaning “this is a tally”).



So far, this has been interpreted as the generation of two-from-one to give a total of three. But it could also express a branching of one-into-two or a division in the ratio of 1:2, which can be expressed as  $\frac{1}{2}$ . In the same way, the first square number  $\overline{\sqcup\sqcup\sqcup}$  could be interpreted as 1:3 or  $\frac{1}{3}$ .

The Rule for Fractional Analysis
$\overline{X} = \frac{1}{X}$

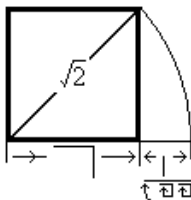
This interpretation is called “fractional analysis” and it is easiest to understand in Brownian notation where a mark over an expression inverts it (Lewin, 2018, pp. 296–301). Thus, where there is a tally expression for two,  $\sqcup\sqcup$ , half is  $\overline{\sqcup\sqcup}$ . Unit fractions are simple tally inversions, while other fractions are expressed as finite continued fractions. Here is a visualization of how this interpretation presents  $\frac{2}{3}$ :

$$\overline{\overline{\sqcup\sqcup\sqcup}} \Rightarrow \frac{1}{\frac{1}{\frac{1}{1+1}} + \frac{1}{1}} \Rightarrow \frac{1}{\frac{1}{1+1} + 1} = \frac{2}{3}$$

Fractional analysis of re-entry expressions results in infinite continued fractions. Consider firstly elementary re-entry as  $1/1/1/1 \dots = 1$ . Thus,  $\overline{\sqcup} = \sqcup$ . However, elementary re-entry in the second degree,  $\overline{\overline{\sqcup}}$ , presents as the infinite continued fraction for the fractional component of the golden mean, which is also  $1/\Phi$ . Here is a visualization of this interpretation of the triangular series:

$$\begin{array}{c} \bullet \\ \swarrow \quad \searrow \\ \bullet \quad \bullet \\ \swarrow \quad \downarrow \quad \searrow \\ \bullet \quad \bullet \quad \bullet \\ \swarrow \quad \downarrow \quad \downarrow \quad \searrow \\ \bullet \quad \bullet \quad \bullet \quad \bullet \\ \vdots \quad \vdots \quad \vdots \quad \vdots \end{array} \rightarrow \frac{1}{\frac{1}{\frac{1}{1+1}} + \frac{1}{1}} \rightarrow \frac{1}{\frac{1}{1+1} + 1} = \frac{1}{1+1} = \frac{1}{\Phi}$$

Fractional analysis of the square series,  $\overline{\overline{\overline{\sqcup\sqcup\sqcup\sqcup}}}$ , gives the continued fraction for the square’s diagonal,  $\sqrt{2}$ .

$$1 + \frac{1}{2 + \frac{1}{2 + \frac{1}{2 + \frac{1}{2 + \dots}}}} = \sqrt{2}$$


This geometric diagram shows how the Brownian expression for square numbers also measures, by another interpretation, the ratio of the square's diagonal to its side. This is just one illustration of the wonderful symmetry of this arithmetic with its geometry. It suffices to say that just as all quadratic irrationals can be expressed as infinite continued fractions, so too can they be expressed by this interpretation in Brownian notation. As for the transcendentals,  $\pi$  and  $e$ , they only require another interpretation. (Lewin, 2020, p. 80). And so we can see how the notation of *Laws of Form* starts to reveal a mensural geometry that is an application of an arithmetic emanation on the principle of one(not-none) or distinction. As Parmenides would say "All is one" but only as Plato might correct: All is one by emanation of its form.

### Acknowledgments

Thanks to those who offered feedback on drafts, including Kelsey Hegarty, Jill Stosic, Andrew McMahon, Joao Leao and Thomas Burwell.

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